UNIVERSIDADE DE ÉVORA

Escola de Ciências e Tecnologia

Departamento de Matemática

Book of Abstracts

Évora-2017

Welcome to WMMGC

Estimados
Participantes, Colegas e Amigos

Este encontro tem como objectivo homenagear a memória da Professora Doutora Graça Carita, natural de Nisa, que faleceu de forma prematura em setembro de 2016. Ao longo da sua vida académica a Professora Doutora Graça Carita destacou-se não só como aluna, como docente da Universidade de Évora, mas ainda como investigadora de nível internacional na área do cálculo das variações. O encontro é aberto a todas as pessoas (independente da sua área de investigação) do Departamento de Matemática da Universidade de Évora, ou não, que de alguma forma privaram com a colega e amiga Graça Carita.

Organizing and Scientific Commitee

Vladimir Goncharov (goncha@uevora.pt), Universidade de Évora

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Russell Alpizar-Jara (alpizar@uevora.pt), Universidade de Évora

Sponsors

O evento tem o apoio da Universidade de Évora, nomeadamente do Departamento de Matemática, da Escola de Ciências e Tecnologia, do Instituto de Investigação e Formação Avançada, do Gabinete de Imagem e Comunicação. E, ainda, do Centro de Investigação em Matemática e Aplicações (CIMA).

In Memoriam

Faleceu no passado dia 26 de setembro de 2016, vítima de doença incurável, a Professora Doutora Graça Carita, matemática portuguesa que se destacou pela sua dedicação integral ao ensino e à matemática. O seu desaparecimento representa enorme perda para o Departamento de Matemática onde lecionava. Natural de Nisa, distrito de Portalegre, a Professora Doutora Graça Carita formou-se com distinção em Matemática Aplicada pela Universidade de Évora. Foi a melhor aluna do seu ano de formação, 1997, especializando-se poucos anos depois com o grau de Mestre, pela mesma universidade, defendendo a tese *Rank-one convexity in the context of lower semicontinuity in the calculus of variations involving 22 symmetric gradients*, obtida sob orientação do Professor Doutor António Ornelas e no mesmo período em que ensinava como Assistente Estagiária.

Com dispensa de serviço docente da sua universidade e uma bolsa de estudo da Fundação para a Ciência e Tecnologia, tomou novo rumo, em direção aos Estados Unidos, integrando-se num dos melhores centros desse país para o estudo do Cálculo das Variações. Na Universidade de Carnegie Mellon foi orientada pela célebre matemática de origem portuguesa Professora Doutora Irene Fonseca e pelo matemático Professor Doutor Giovanni Leoni. A sua tese de Doutoramento, o qual obteve em 2007, intitula-se *Relaxation in SBVp for constrained valued-fields*. Os trabalhos de Graça Carita foram crescendo, atingindo sempre enorme rigor e profundidade, emprestando muito saber e prestígio ao Departamento de Matemática da Universidade de Évora. Nos seus últimos artigos diversificou os temas em colaboração com colegas italianos e portugueses de outras universidades.

A sua postura exigente por melhores ensino e investigação, a par da natural afabilidade no convívio com alunos e colegas, fica na memória de todos os que a conheceram durante a sua vida académica, tão penosamente encurtada. A colega Graça Carita deixa grande saudade entre amigos e familiares, assim como constitui dolorosa perda para o Departamento de Matemática da Universidade de Évora, nomeadamente para o grupo de Análise Matemática. O vigoroso exemplo da formação de uma docente e investigadora de entre as jovens do seu país foi atingido, pelo que será certamente lembrado e seguido por novas gerações.

Poem to Graça Carita

Tu, não!
(Lembrando a Graça)

Parou na ponte
e para um mar negro
do alto se atirou…

Tu, não!
Agarrada sempre aos ossos,
tu, não!

Às correntes da vida
uma esperança presa
mesmo finos os fios
ali sentindo sua vida…

Tu, não!
Só quando dos ossos
a vida te foi arrancada!

Desamarrada da vida,
só amarrada à morte!

Só presa pela morte,
livre tu não te foste!
Tu, não!

Ficaste,
livre por aqui vagando,
ficando…

José Rodrigues Dias, 2016-12-12

Scientific Program

Encontro de Matemática em Memória de Graça Carita

Universidade de Évora, Colégio Luís António Verney, Anfiteatro nº1

Évora, Portugal, Março 3-4, 2017

Programa

Sessão de Abertura (10:00-11:00)	
Sexta-feira, 3 de Março de 2017	
10:00-11:00	Início dos trabalhos Universidade de Évora, Profs Irene Fonseca e Giovanni Leoni (Carnegie Mellon University, USA), representante de família, colegas e amigos.
11:00-11:25	**Coffee Break**
Sessão I (11:30-13:30)	
Chair: Russel Alpizar-Jara (Universidade de Évora, Portugal)	
11:30-11:55	**Elvira Zappale** (Universitá di Salerno, Itália) Some mathematical results for optimal design problems
12:00-12:25	**Ana Cristina Barroso** (Universidade de Lisboa, Portugal) A model for phase transitions with competing terms
12:30-12:55	**Feliz Minhós** (Universidade de Évora, Portugal) Solvability of three-point third-order coupled systems
13:00-14:30	**Almoço, refeitório do Colégio Luís António Verney**
Sessão II (14:30-18:00)	
Chair: Elvira Zappale (Universitá di Salerno, Itália)	
14:30-14:55	**Eugénio Rocha** (Universidade de Aveiro, Portugal) Existence of solutions of quasilinear systems with measure data
15:00-15:25	**Luís Bandeira** (Universidade de Évora, Portugal) Quasiconvexity, rank-one convexity and non-negative polynomials
15:30-15:55	**Carlos Braumann** (Universidade de Évora, Portugal) Life and chance
16:00-16:25	**Coffee Break**
16:30-16:55	**Margarida Baía** (Instituto Superior Técnico, Portugal) A degenerate elliptic-parabolic system arising in competitive contaminant transport
17:00-17:25	**Pedro Marques** (Universidade de Évora, Portugal) The rank of a homogeneous polynomial

17:30-17:55	**Joaquim Correia** (Universidade de Évora, Portugal) Burgers-KdV type equations with saturating dissipation
20:00	**Jantar do Encontro no Restaurante S. Luís** **(local de encontro pelas 19:45 no portão de entrada do Colégio Luís António Verney)**

Sessão III (10:00-13:00)	
Sábado, 4 de Março de 2017	
Chair: Vladimir Goncharov (Universidade de Évora, Portugal)	
10:00-10:25	**Luísa Mascarenhas** (Universidade Nova de Lisboa, Portugal) A chromoticity-brightness model for color images denoising
10:30-10:55	**Ana Margarida Ribeiro** (Universidade Nova de Lisboa, Portugal) Supremal functionals: existence of minimizers in the scalar setting and obstacles when moving to the vectorial case
11:00-11:25	**José Matias** (Instituto Superior Técnico, Portugal) Dimension reduction in the context of first-order structured deformations
11:30-11:55	**Coffee Break**
12:00-12:25	**Pedro Areias** (Universidade de Évora, Portugal) Phase field Analyis of finite-strain plates and shells including element subdivision
12:30-12:55	**Vladimir Goncharov** (Universidade de Évora, Portugal) Variational problema of the plastic surgery
13:00	**Encerramento**

Abstracts

Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

Some mathematical results for optimal design problems

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ABSTRACT

I will describe some variational results (most of them obtained in collaboration with Graça Carita) about nonlinear optimal design. Asymptotic analysis and integral representation for the limit models will be deduced.

Keywords: Optimal design, functions of bounded variations.

References

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Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

A model for phase transitions with competing terms

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ABSTRACT

Given a small parameter $\varepsilon > 0$, we study the asymptotic behaviour of the family of energy functionals

$$E_\varepsilon(u,\rho) = \frac{1}{\varepsilon}\int_\Omega f(u(x), \varepsilon\nabla u(x), \varepsilon\rho(x))\,dx,$$

where $u \in W^{1,p}(\Omega;^d)$ represents the vector-valued fluid density of d fluids present in a container $\Omega \subset^N$ and ρ is a scalar field which interplays with ∇u in the formation of interfaces.

Without imposing any convexity assumptions on the density function f, and under the volume constraints $\int_\Omega u(x)\,dx = V_f$ and $\int_\Omega \rho(x)\,dx = V_s$, we identified the Γ-limit of the functionals $E_\varepsilon(u,\rho)$ relative to the convergence in an appropriate space.

This is joint work with M. Baía and J. Matias.

Keywords: two-phase transition problems, functions of bounded variation, Γ-convergence, surfactants

Tribute: I met Graça in 1997, when she was still a student, at a Summer School in Coimbra. A few years later, like I had also done, she went to Carnegie Mellon University to obtain her Ph.D under the supervision of Irene Fonseca (in her case, also co-advised by Giovanni Leoni). As we ended up working in the same area of Mathematics, our paths crossed many times and we went from being colleagues to becoming friends. Graça was a very easy person to like. She was generous, welcoming and helpful towards others, with a sweet side, but also determined and strong willed. She had many hurdles in her too short life, I admire the tenacity and courage with which she faced them. I think of her often and I miss her dearly.

Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

Acknowledgements

A.C.B belongs to the Centro de Matemática, Aplicações Fundamentais e Investigação Operacional, Universidade de Lisboa, a research centre supported by FCT (Fundação para a Ciência e a Tecnologia, Portugal).

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Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

Solvability of three-point third-order coupled systems

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ABSTRACT

In this talk we present sufficient conditions for the solvability of the third order three point boundary value problem

$$\begin{cases} -u'''(t) = f(t,\, v(t),\, v'(t)) \\ -v'''(t) = h(t,\, u(t),\, u'(t)) \\ u(0) = u'(0) = 0, u'(1) = \alpha u'(\eta) \\ v(0) = v'(0) = 0, v'(1) = \alpha v'(\eta), \end{cases}$$

The non-negative continuous functions f and h verify adequate superlinear and sublinear conditions, $0 < \eta < 1$ and the parameter α such that $1 < \alpha < (1/\eta)$. The arguments apply Green's function associated to the linear problem and the Guo–Krasnosel'skiĭ theorem of compression-expansion cones. The dependence on the first derivatives is overcome by the construction of an adequate cone and suitable conditions of superlinearity/sublinearity near 0 and $+\infty$.

Keywords: Coupled systems, Green functions, Guo–Krasnosel'skii fixed-point in cones, positive solution.

Tribute: Dedicated to the memory of Graça Carita

Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

Existence of solutions of quasilinear systems with measure data

Francesco Leonetti[1], Eugénio Rocha[2]* and Vasile Staicu[2]

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ABSTRACT

We study the existence of solutions of quasilinear elliptic systems involving N equations and a measure on the right hand side, with the form

$$\begin{cases} -\sum_{i=1}^{n} \frac{\partial}{\partial x_i} \left(\sum_{\beta=1}^{N} \sum_{j=1}^{n} a_{i,j}^{\alpha,\beta}(x,u) \frac{\partial}{\partial x_j} u^{\beta} \right) = \mu^{\alpha} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\alpha \in \{1, \dots, N\}$ is the equation index, Ω is an open bounded subset of $\mathbb{R}^n$, $u : \Omega \to \mathbb{R}^N$ and μ is a finite Randon measure on $\mathbb{R}^n$ with values into $\mathbb{R}^N$. Existence of a solution is proved for two different sets of assumptions on the coefficients $a_{i,j}^{\alpha,\beta}$. Examples are provided that satisfy our conditions, but do not satisfy conditions required on previous works on this matter. This is a joint work with Francesco Leonetti and Vasile Staicu.

Keywords: elliptic systems, existence of solutions, measure data

Tribute: Im memory of Graça Carita for her contributions as scientist, teacher and person.

Acknowledgements

Partial support by Portuguese funds through CIDMA - Center for Research and Development in Mathematics and Applications, and the Portuguese Foundation for Science and Technology (FCT), within the project PEst-OE/MAT/UI4106/2014.

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Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

Quasiconvexity, rank-one convexity and non-negative polynomials

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ABSTRACT

We report our latest work about non-negativeness of polynomials and the main necessary and sufficient conditions for weak lower semicontinuity of integral functionals in vector calculus of variations. The celebrated theorem about sum of squares and non-negativeness of polynomials, due by David Hilbert, plays a special role, providing new tools to investigate rank-one convexity of functions defined on 2×2-matrices. For these results, we explore some consequences and examples.

Keywords: Quasiconvexity, rank-one convexity, non-negative polynomials and sums of squares

Acknowledgements

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Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

Life and chance

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ABSTRACT

Life and chance are indissociable. The dynamics of biological phenomena is influenced by intrinsic and environmental factors in which chance has a relevant role. Thus the need to use stochastic models, in our case stochastic differential equations (SDE), to describe such phenomena and make predictions. We will describe some uses of SDE on population growth and in determining extinction risks, with applications to the optimal use of living resources, to wildlife preservation and to food safety. We will also mention some other uses of SDE in individual growth of farm animals and in demography and mention some useful applications. These models, that were born from Physics and are nowadays the backbone of financial assets modeling, have applications in many other areas. They are indeed a good paradigm of cross-fertilization among scientific areas since many of the most relevant mathematical developments were driven by applications.

Keywords: Stochastic differential equations; randomly varying environments; extinction time; applications in Life Sciences.

Tribute: In this homage to our colleague Graça Carita we celebrate life, her life, shorter by chance than we would like but a great gift in personal and institutional terms that will endure by her achievements and in our memories.

Acknowledgements

The researcher belongs to the Centro de Investigaçãoo em Matemática e Aplicações, Universidade de Évora, Project UID/MAT/04674/2013, a researh centre supported by FCT (Fundaçãoo para a Ciência e a Tecnologia, Portugal).

Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

A degenerate elliptic-parabolic system arising in competitive contaminant transport

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ABSTRACT

In this work we study a coupled system of degenerate and nonlinear partial differential equations governing the transport of reactive solutes in groundwater. We show that the system admits a unique weak solution provided the nonlinear adsorption isotherm associated with the reaction process satisfies certain physically reasonable structural conditions. We conclude, moreover, that the solute concentrations stay non-negative if the source term is componentwise non-negative and investigate numerically the finite speed of propagation of compactly supported initial concentrations, in a two-component test case.

Keywords: Degenerate elliptic-parabolic, system of PDEs.

Tribute: To Graça as a "Thinkforyourselfees" kind friend.

The Yesees said yes to anything, that anyone suggested.
The Noees said no to everything, unless it was proven and tested.
So the Yesees all died of much too much, and the Noees all died of fright,
But somehow I think the Thinkforyourselfees, all came out all right.[1]
(Yesees and Noees , S. Silverstein)

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Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

The rank of a homogeneous polynomial

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ABSTRACT

The rank of a homogeneous polynomial f of degree d is the minimal number of summands in a presentation of f as a sum

$$f = l_1^d + \cdots + l_r^d,$$

where $l_1, \ldots, l_r$ are linear forms. It has a close relation to problems in complexity theory and very interesting interpretations in geometry, namely in the study of secant varieties to the Veronese variety. I will make a short introduction to this subject, and report on the main known results and open questions.

Keywords: Rank of a homogeneous polynomial, apolarity.

Tribute: In memory of Graça Carita. We will miss her honest and free-spirit thinking.

Acknowledgements

The researcher belongs to the Centro de Investigação em Matemática e Aplicações, Universidade de Évora, a researh centre supported by FCT (Fundação para a Ciência e a Tecnologia, Portugal).

Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

Burgers-KdV type equations with saturating dissipation

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ABSTRACT

We are regarding to convective-dissipative-dispersive flux model problems in media without strong spacial or temporal inhomogeneities of the form $u_t + f(u)_x = \varepsilon\, b(u_x)_x - \delta\, g(u_{xx})_x$. Proper Burgers and KdV equations corresponds respectively to $\delta = 0$ and $\varepsilon = 0$ with $f(u) = u^2/2$ and linear terms $b(\cdot)$ and $g(\cdot)$. Those linear terms becomes from the assumption of small gradients and hessians when the continuum equations are derived from the physical microscopic systems and the flux functional expansion in polynomials is truncated. Linear flux gradient or hessian relations provides infinite flux responses to sharp interfaces, while the physical response is clearly finite. Therefore we are considering here nonlinear flux functions $b(\cdot)$ and $g(\cdot)$ that saturate at infinite. We expect the solution to the Cauchy problem to be smooth or discontinuous according to a critical threshold of the upstream-downstream transition. (This work is a consequence of the collaboration we began on this subject with Michael Grinfeld from the Dept. of Math. & Stat. of Strathclyde University, Glasgow, UK.)

Keywords: Burgers-KdV type equations, saturating dissipation-dispersion, scalar conservation laws, entropy solutions, Young measures, measure-valued solutions

Acknowledgements

This research is partially supported by FCT (Portugal) under the contract programme with CIMA UID/MAT/04674/2013 and by the CNRS (France) – FCT (Portugal) PICS project no. 40436 “Modelling, Analysis and Simulations of Coagulant Fluids”

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Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

A chromaticity-brightness model for color images denoising

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ABSTRACT

In this talk a variational model for imaging segmentation and denoising color images is proposed. The model combines Meyer's "u+v" decomposition with a chromaticity-brightness framework and is expressed by a minimization of energy integral functionals depending on a small parameter $\epsilon > 0$. The asymptotic behavior as $\epsilon \to 0^+$ is characterized, and convergence of infima, almost minimizers, and energies are established. In particular, an integral representation of the lower semicontinuous envelope, with respect to the L^1-norm, of functionals with linear growth and defined for maps taking values on a certain compact manifold is provided. This study escapes the realm of previous results since the underlying manifold has boundary, and the integrand and its recession function fail to satisfy hypotheses commonly assumed in the literature. The main tools are Γ-convergence and relaxation techniques.

Keywords: imaging denoising, color images, chromaticity-brightness decomposition, Γ-convergence, relaxation BV functions, manifold constraints.

Tribute: À Graça.

Acknowledgements

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Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

Supremal functionals: existence of minimizers in the scalar setting and obstacles when moving to the vectorial case

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ABSTRACT

In this talk, it will be discussed necessary and sufficient conditions for existence of solutions to the problem

$$\inf \left\{ \operatorname*{ess\,sup}_{x\in\Omega} f(\nabla u(x)) : u \in u_0 + W_0^{1,\infty}(\Omega;\mathbb{R}^N) \right\}.$$

In the scalar setting, $N = 1$, the interesting case is when the supremand f is not necessarily level convex. In this case, necessary and sufficient conditions are obtained through a comparison with a related level convex problem and written in terms of a differential inclusion involving the boundary datum. For $N > 1$, we analyse the obstacles found to apply the same procedure. This is a joint work with E. Zappale published in [1].

Keywords: Minimizers, supremal functionals.

Tribute: While life was too short and demanding for Graça, let her memory inspire on us the same strength and courage to face the challenges in our lives.

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Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

Dimension reduction and structured deformations

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ABSTRACT

We tackle the problem of dimension reduction in the context of structured deformations. Our aim is to apply both procedures of dimension reduction and relaxation of energy for structured deformation to a three-dimensional energy defined on a thinning domain. We shall apply the two processes one after the other, exchanging the order, and therefore obtain two pairs of relaxed bulk and surface energy. For the specific choice of an initial energy including only the surface term, we compute the energy densities explicitly and conclude that they are the same, independently of the order of the relaxation processes. Moreover, for this specific choice of initial energy, we compare our results with those obtained when the limiting process of dimension reduction and passage to the structured deformation is carried out at the same time.

Keywords: Dimension reduction, Structured Deformations, Relaxation.

Tribute: In loving memory of Graça. Despite her terrible illness, she never gave up and tried to pursue this work until the end.

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Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

Phase-field analysis of finite-strain plates and shells including element subdivision

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ABSTRACT

In this presentation, the theme of fracture of finite-strain plates and shells is introduced. We present a phase-field model of crack regularization and introduce a new staggered algorithm for elastic and elasto-plastic materials. To account for correct fracture behavior in bending, two independent phase-fields are used, corresponding to the lower and upper faces of the shell. This is shown to provide a realistic behavior in bending-dominated problems, here illustrated in classical beam and plate problems. Finite strain behavior for both elastic and elasto-plastic constitutive laws is made compatible with the phase-field model by use of a consistent updated-Lagrangian algorithm. To guarantee sufficient resolution in the definition of the crack paths, a local remeshing algorithm based on the phase-field values at the lower and upper shell faces is introduced. In this local remeshing algorithm, two stages are used: edge-based element subdivision and node repositioning. Pioneering work in the analysis of shells using phase-field developments was performed by T. Rabczuk's group [2]. In essence, phase-field models introduce diffuse cracks (diffusivity depends on a length parameter) which are energy-equivalent to the original sharp cracks. In addition, in the original brittle formulation (cf. [5, 3, 4]), the solution converges to the A.A. Griffith's model [6] as the length parameter tends to zero. A formal proof of convergence of diffuse (regularized) solutions to the sharp original formulation was established by Ambrosio and Tortorelli [1]. The phase-field model is able to represent crack initiation and propagation, coalescence and branching, which are intricately dealt with enrichment methods.

Keywords: Phase-field model for fracture, shells, engineering application of Γ-convergence.

Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

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Workshop on Mathematics in memory of Graça Carita
Évora, Portugal, March 3-4, 2017

Variational problem of the plastic surgery

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ABSTRACT

Our talk is devoted to some results concerning the problem studied jointly with Graça two years ago, which will be published soon (in the June, 2017, see [1]) being one of the latest her works. We consider the variational problem arising in plastic surgery, namely, during the suturing procedure in the breast reduction surgery. This problem consists in minimization of a polyconvex integral functional under, in particular, unusual "knitting" boundary conditions, which model the suturing of one cut piece of the breast to another one. We prove existence of a minimizer in this variational problem (the part of research by Graça Carita and V.G.) and deduce some necessary condition of optimality, which helps to isolate a minimizer.

Keywords: Calculus of Variations, polyconvex integrand, coercivity assumptions, trace operator, knitting boundary conditions.

Tribute: Our talk is dedicated to the bright memory of Graça Carita, very good friend and talented hopeful mathematician, feeling enormous gratitude and happiness to work in teaching and research and to communicate with her. She will always remain in our hearts.

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